The Relationship Between Dual Mode Cavity Cross-Coupling and Waveguide Polarizers

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Abstract—**Cross-coupling in dual-mode cavity filters may be obtained by introducing an asymmetry within the cavity cross section at an angle of 45° to the two orthogonal modes. This paper presents a novel formula relating the resulting cross coupling coefficient between the orthogonal resonances to the polarization of a waveguide polarizer. Previous theories for such polarizers may then be applied directly to the dual mode filter situation. Formulas enabling the dimensions of the asymmetries for required coupling coefficients are presented for square and circular waveguide cross sections.**

I. Introduction

DUAL MODE CAVITY FILTERS have been widely used in communications satellites since it was pointed out that this type of filter simplifies the realization of cross-coupling between electrically nonadjacent resonances, which may actually occur in physically adjacent cavities [1]. Further details and references may be found in [2]. Such filters are now being used also for some specialized nonsatellite applications.

The usual way to couple between the orthogonal dual modes in a given cavity is by adding a screw at 45° with respect to the electric fields of the two modes. This method has disadvantages due to the large screw penetration often required, resulting in field distortions, reduction in unloaded Q, and reduced power handling capability. Also field theory has difficulty predicting the amount of screw penetration required to realize a given coefficient of coupling between the orthogonal modes.

A solution to this problem has been used at Hughes Aircraft Company for several years [3], and proposed independently by Fiedziuszko for dual mode microstrip cavities [4]. Later this method was extended to dual mode waveguide cavities [5]. In the case of the square cavities discussed in these two papers [4], [5], the mode coupling is by means of a portion removed from one corner of the cavity.

Here it is observed that this type of mode coupling was introduced much earlier for a completely different application, namely the design of waveguide polarizers, e.g., [6] and [7]. The latter reference [7] treats a square waveguide polarizer having two diagonally opposite corners cut away as shown in Fig. 1, as contrasted with the single cut corner of [4] or the single rectangular cut-away of [5]. It will be shown that the previous theories relating to waveguide polarizers may be applied to the design of the mode coupling in cavities. This then leads to the interesting question of whether there is a relationship between the polarization properties of the waveguide polarizer and the cavity mode coupling. The answer rather obviously is that there is such a relationship, which will be given in this paper.

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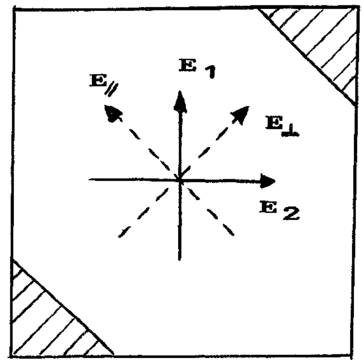

Fig. 1. Cross section of square waveguide with cut corners or "flats."

Since [7] is a somewhat obscure reference some of the material on the double-cut corner polarizer will be repeated herein, with the opportunity taken to make some upgrades and corrections. The method is applied also to the case of asymmetries in the form of "flats" on waveguides of circular cross section, which is more widely used in filters than square cavities. Flats on dual-mode dielectric resonators may be treated similarly [8].

II. The Relationship Between Coupling Coefficient and Polarizability

A cross section of the square waveguide polarizer having diagonally opposite corners cut off is shown in Fig. 1. In a filter the orthogonal modes are polarized with E vectors indicated as E_1 and E_2, i.e., the flats are at an angle of 45° to both vectors. In a waveguide polarizer if the incident field is E_1 then the polarizer may be designed to rotate the plane of polarization through 90° to give an output polarization of E_2.

The incident wave may be resolved into two components $E_{\|}$ and $E_{\perp}$ as shown in Fig. 1. In the case of the polarizer the difference in phase shift over a length ℓ of waveguide between these two symmetric modes is

$$\Delta\theta = \theta_{\|} - \theta_{\perp} = 2\pi\ell(1/\lambda_{g_{\|}} - 1/\lambda_{g_{\perp}}). \tag{1}$$

The guide wavelengths $\lambda_{g_{\|}}$ and $\lambda_{g_{\perp}}$ in (1) are to be derived from a calculation of the cut-off wavelengths of the symmetric modes.

In the case of the waveguide cavity, it is desired to introduce a coefficient of coupling k between the modes E_1 and E_2. This

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may be expressed in terms of the resonant frequencies of the orthogonal modes E and E as [5]

$$k = \frac{f_{\parallel}^2 - f_{\perp}^2}{f_{\parallel}^2 + f_{\perp}^2} = \frac{\Delta f}{f} \tag{2}$$

where an approximation valid for the narrow to moderate bandwidths realizable for dual mode filters has been made. This may be expressed in terms of the difference between the guide wavelengths of the symmetric modes using the formula

$$\frac{df}{d\lambda_g} = -\left(\frac{\lambda}{\lambda_g}\right)^2 \frac{f}{\lambda_g} \tag{3}$$

giving (dropping the unimportant negative sign)

$$k = \left(\frac{\lambda}{\lambda_g}\right)^2 \frac{\Delta\lambda_g}{\lambda_g}. \tag{4}$$

The desired relationship between $\Delta\theta$ and k is now established by equating $\Delta\lambda_g$ to the difference of guide wavelengths given by (1) for the polarizer, which for cases where the flats are small, resulting in a relatively small difference between the guide wavelengths of the modes, may be closely approximated as

$$\Delta\theta = \frac{2\pi\ell}{\lambda_g}\frac{\Delta\lambda_g}{\lambda_g}. \tag{5}$$

Elimination of $\Delta\lambda_g$ from (4) and (5) leads to an expression relating the polarization $\Delta\theta$ required to the coefficient of coupling k as

$$\Delta\theta = \frac{2\pi\ell}{\lambda_g}\left(\frac{\lambda_g}{\lambda}\right)^2 \cdot k \tag{6}$$

where ℓ is the length of the waveguide cavity, and the guide wavelength is defined for the unperturbed cavity. Of course the more accurate expressions (1) and (2) may be used, but (6) will be sufficiently accurate for all normal narrow band dual mode filters.

It is useful to re-express (5) using the relationship

$$\frac{\Delta\lambda_g}{\Delta\lambda_c} = \left(\frac{\lambda_g}{\lambda_c}\right)^3 \tag{7}$$

to give

$$\Delta\theta = \frac{2\pi\ell}{\lambda_g}\left(\frac{\lambda_g}{\lambda_c}\right)^2 \cdot \frac{\Delta\lambda_c}{\lambda_c}. \tag{8}$$

Hence (6) may be re-expressed to give the following useful relationship between k and the polarizability expressed in terms of $\Delta\lambda_c/\lambda_c$, i.e.,

$$\frac{\Delta\lambda_c}{\lambda_c} = \frac{\lambda_{c_{\parallel}} - \lambda_{c_{\perp}}}{\lambda_c} = \left(\frac{\lambda_c}{\lambda}\right)^2 \cdot k. \tag{9}$$

It will be noted that the cavity polarizability is very small for a dual mode cavity compared with that required for a 90° waveguide polarizer because of the the small value of coupling coefficient k, yet the electric vector is rotated through 90° in the cavity. The physical reason for this is that the wave in a filter cavity is delayed so that the field vector has more time available to be rotated. In fact it is simple to show that the polarizability required is inversely proportional to the group delay of the dual mode filter.

Another interesting point is that (9) implies that the polarizability as defined by the waveguide cross sectional dimensions is independent of the cavity length. Hence it will be the same for all TE_{11n}-mode filters independent of n. This too has a simple physical explanation, namely that the coupling coefficients of a TE_{11n}-mode filter are n times larger than those for a TE_{111}-mode filter of the same bandwidth. Hence the polarizabilities per unit length of the waveguide is independent of n since the waveguide length is proportional to n.

The small values of polarizability required enables dimensions for the polarizing structure to be derived using simple closed form expressions obtained from perturbation theory, as follows.

III. Double Cut Corner Polarizer in Square Waveguide

The theory given in [7] is now reviewed with corrections and improvements. Fig. 2 indicates the novel technique adopted to simplify the field problem, whereby the awkward diagonal regions of the original waveguide cross section are transformed into two simpler square waveguide problems. The field patterns of each mode, $E_{\parallel}$ and $E_{\perp}$, are shown to be identical to the TE_{11} waveguides illustrated, $E_{\parallel}$ having 4 corners removed (or 4 metallic bars inserted), while $E_{\perp}$ is perturbed by a square coaxial bar. The effect of these bars is then calculated using perturbation theory. A waveguide operating at its cut-off frequency is considered to be in resonance, a principle often used for example to calculate the cut-off frequency of ridge waveguide using the condition for transverse resonance [9].

Considering a unit length of the waveguide of Fig. 2(b), the change in transverse resonant frequency at cut-off arising from the perturbation caused by the four bars is obtained from Slater's perturbation theorem [10] as

$$\frac{\Delta f}{f_o} = \frac{\Delta U_E - \Delta U_H}{2U_o} \tag{10}$$

where the unperturbed resonant frequency is $f_o, \Delta U_E$ and ΔU_H are the peak electric and magnetic fields over the perturbed region, and U_o is the total stored energy.

1) Parallel Field Component: In Fig. 2(b) the E field may be neglected in the region of the perturbations, giving

$$U_H = \frac{1}{2}\mu|H_z|^2\Delta V \tag{11}$$

where ΔV is the perturbing volume. This is the formula for the stored energy given in [7], but it is equally valid to take the stored energy of the transverse electric field, which leads to an identical result. The reason for the identity of the two approaches is that energy is transferred continuously between the transverse electric and longitudinal magnetic fields. Since the waveguide is at resonance at cut-off, the energies stored in the two fields are equal.

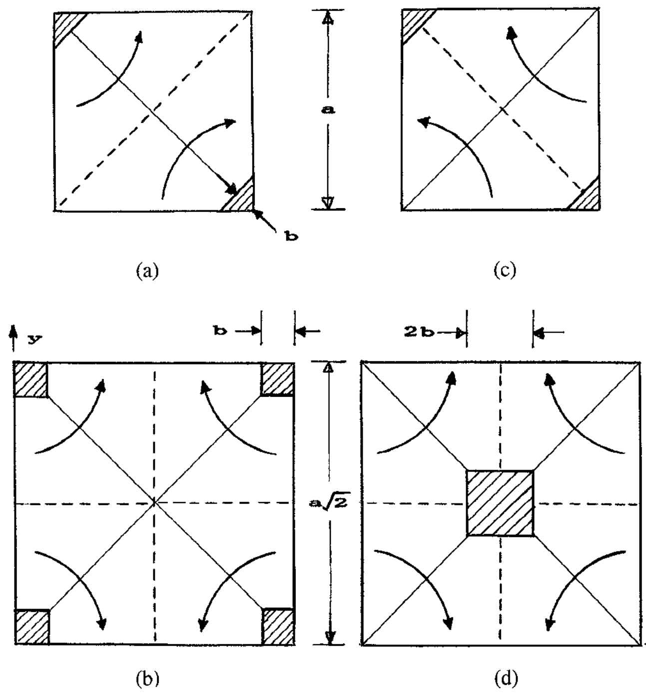


Fig. 2. Illustrating the transformation of the original field problems into simpler regular field problems: (a) The $E_{\|}$ mode of the polarizer showing the electric and magnetic walls. (b) The TE_{11} mode perturbed by four square corner bars—note the triangular sections corresponding to those in (a). (c) The $E_{\perp}$ mode of the polarizer. (d) The TE_{11} mode perturbed by a square coaxial bar.

With suitable choice of origin the H_z field of the TE_{11} mode is

$$H_z = B\cos\left(\frac{\pi x}{\sqrt{2}a}\right)\cos\left(\frac{\pi y}{\sqrt{2}a}\right) \tag{12}$$

where B is a constant. At this point the original report [7] makes an approximation by taking the field to be uniform over the volume, but there is no need for this since the integral has a simple exact solution. The energy displaced in one of the rectangular bar regions of Fig. 2(b) is

$$\begin{aligned} U_H &= \frac{1}{2}\mu B^2 \int_0^b \int_0^b \cos^2\left(\frac{\pi x}{\sqrt{2}a}\right)\cos^2\left(\frac{\pi y}{\sqrt{2}a}\right) dx\,dy \\ &= \frac{1}{2}\mu B^2 \frac{1}{4}\left[b + \frac{2a}{\sqrt{2}\pi}\sin\left(\frac{2\pi b}{\sqrt{2}a}\right)\right]^2. \end{aligned} \tag{13}$$

This value will be multiplied by 4 to give the total energy displaced in the structure of Fig. 2(b) since there are 4 bars. The total energy stored at resonance is given by

$$U_o = \frac{1}{2}\mu \int_0^{a\sqrt{2}} \int_0^{a\sqrt{2}} |H_z|^2\, dx\,dy = \frac{1}{4}\mu B^2 a^2. \tag{14}$$

Hence applying (10) the shift of cut-off frequency is given by

$$\frac{\Delta f_{\|}}{f} = -\frac{\Delta U_{H_{\|}}}{2U} = \left(\frac{b}{a}\right)^2 \left[1 + \frac{\sin\left(\frac{\sqrt{2}\pi b}{a}\right)}{\frac{\sqrt{2}\pi b}{a}}\right]^2. \tag{15}$$

The shift in cut-off wavelength is derived by applying the equation

$$\frac{\Delta f}{f_o} = -\frac{\Delta\lambda}{\lambda_o} \tag{16}$$

where

$$\Delta\lambda = \lambda_{c_{\|}} - \lambda_{c_{\|}} \quad ; \quad \lambda_o = \lambda_{c_{\|}} \tag{17}$$

and $\lambda_{c_{\|}}$ is the cut-off wavelength of the perturbed TE_{11} mode and λ_{c11} is the cut-off wavelength of the unperturbed TE_{11} mode. This cut off wavelength is twice the waveguide width or height, i.e.,

$$\lambda_{c11} = 2a. \tag{18}$$

Using (15)–(18) the cut-off wavelength of the $E_{\|}$ mode of Fig. 2(a) is given by

Parallel field

$$\frac{\lambda_{c_{\|}}}{2a} = 1 - \left(\frac{b}{a}\right)^2 \left[1 + \frac{\sin\frac{\sqrt{2}\pi b}{a}}{\frac{\sqrt{2}\pi b}{a}}\right]^2. \tag{19}$$

2) Perpendicular Field Component: The theory here is similar to that outlined for the parallel field component, except that now the energy displaced by the central coaxial bar in Fig. 2(d) is almost entirely electric, i.e.,

$$U_E = \frac{1}{2}\varepsilon[E_x^2 + E_y^2]\Delta V \tag{20}$$

where

$$E_x = -E_y = j\eta\frac{\sqrt{2}}{2}B\cos\left(\frac{\pi x}{\sqrt{2}a}\right)\sin\left(\frac{\pi y}{\sqrt{2}b}\right) \tag{21}$$

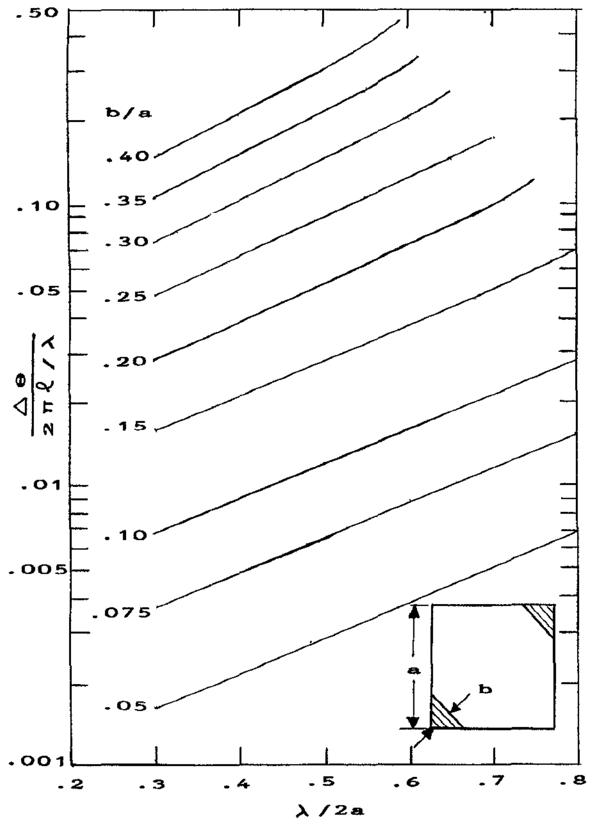


Fig. 3. Normalized differential phase shift $\Delta\theta/(2\pi\ell/\lambda)$ vs $\lambda/2a$ with b/a as a parameter.

evaluated in the region $(a\sqrt{2}-b) < x, y < (a\sqrt{2}+b)$. The energy displaced by the bar is therefore

$$U_E = -\frac{1}{4}\mu B^2 \int_{\frac{a}{\sqrt{2}}-b}^{\frac{a}{\sqrt{2}}+b} \cos^2\left(\frac{\pi x}{\sqrt{2}a}\right) dx \cdot \int_{\frac{a}{\sqrt{2}}-b}^{\frac{a}{\sqrt{2}}+b} \sin^2\left(\frac{\pi y}{\sqrt{2}b}\right) dy. \quad (22)$$

Reasoning similar that described for the parallel field leads to the final expression for the cut-off wavelength for the perpendicular field as

Perpendicular field

$$\frac{\lambda_{c_\perp}}{2a} = 1 + \frac{\Delta U_{E_\perp}}{2U_o} = 1 + \left(\frac{b}{a}\right)^2 \left[1 - \frac{\sin\left(\frac{\sqrt{2}\pi b}{a}\right)}{\frac{\sqrt{2}\pi b}{a}}\right] \cdot \left[0.5 + \frac{\sin\left(\frac{\sqrt{2}\pi b}{a}\right)}{\frac{\sqrt{2}\pi b}{a}}\right]. \quad (23)$$

Equations (19) and (23) agree closely with those given in [7] for small values of the "flat" parameter b, but the plots of normalized differential phase shift [7, Fig. 4] are rather difficult to read and appear to contain errors. A corrected series of plots is given in Fig. 3.

A. Experimental Results

The validity of the theory has been checked by comparison with measured results reported for a waveguide polarizer reported in [11, p. 60] for a polarizer with linearly tapered transitions to match into a uniform section with cut corners. The polarization in the tapered regions are derived by simple integration over the length of the tapers. A comparison between the theory and the measurements is shown in Fig. 4. The two curves may be brought into coincidence by reducing the b dimension of the flats from 0.1525 in. to 0.151 in., indicating agreement between the simple perturbation theory and measurements within practical tolerances.

A second test has been carried out using the Hewlett-Packard numerical electromagnetic field software package HFSS for the cross sections indicated in Table I, which compares the results obtained for the shift in cut-off frequencies for the parallel and perpendicular field orientations with the present theory. The deviation increases for larger values of b where the perturbation theory becomes less accurate, though still very acceptable, and the accuracy is excellent for the relatively small flat dimensions encountered in dual mode filters.

IV. Circular Waveguide Polarizer

The type of polarizer considered here is a length of cylindrical waveguide of circular cross section operating in the TE_{11} mode with flats on either one surface or on opposite surfaces as shown in Fig. 5. The radius of the waveguide is a, and the flat is defined by its maximum thickness t. The characteristics of the polarizer are determined if the cut-off frequencies of the two normally degenerate modes $E_{\|}$ and $E_{\perp}$ are known, similarly to the square waveguide case treated in Section III.

This problem was solved originally by Pyle and Angley [12] by a numerical technique. However, application of the perturbation theory as described in Section III gives the results in the form of simple closed-form equations, with quite good agreement with the earlier results [12].

The fields in the unperturbed guide are given by [13] in the case of the TE_{11} or H_{11} mode with $m = 1$ and $\chi = 1.841$ as

$$\begin{aligned} E_r &= BV[J_1(\chi r/a)/r]\sin\theta \\ E_\theta &= BV\chi[J_1'(\chi r/a)/a]\cos\theta \\ E_z &= 0 \\ H_r &= -BI\chi[J_1'(\chi r/a)/a]\cos\theta \\ H_0 &= BI[J_1(\chi r/a)/r]\sin\theta \\ H_z &= -jB\sqrt{\frac{\varepsilon}{\mu}}V\frac{\lambda}{2\pi a}\chi^2[J_1(\chi r/a)/a]\cos\theta \end{aligned} \quad (24)$$

where B is a constant given by

$$B = \sqrt{(2/\pi)}/\left[\sqrt{\chi^2-1}\cdot J_1(\chi)\right] \quad (25)$$

and V and I are equivalent transmission line peak voltages and currents. where

$$V/I = \sqrt{\mu/\varepsilon}. \quad (26)$$

Note that [13] uses a non standard notation for the impedance of free space as compared with modern terminology, i.e., η in [13] is defined as $\sqrt{\varepsilon/\mu}$ instead of the inverse.

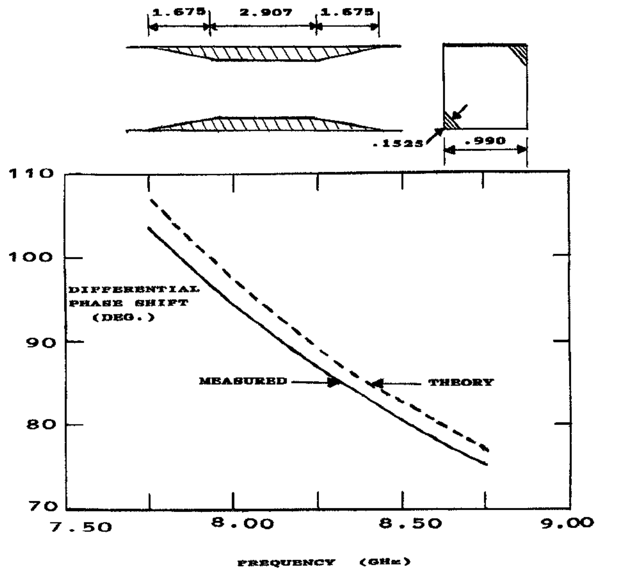
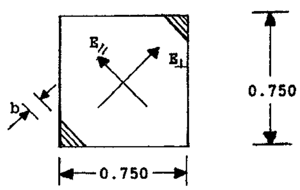


Fig. 4. Comparison of theory with measured performance of MIT polarizer [11, p. 60].

TABLE I
COMPARISON OF RESULTS OBTAINED FROM THE HFSS PROGRAM WITH THE PERTURBATION THEORY



	Cut-off frequencies and frequency split f_c					
	HFSS			Perturbation Theory		
b (in.)	$E_{//}$ (MHz)	$E_{\perp}$ (MHz)	Δf_c (MHz)	$E_{//}$ (MHz)	$E_{\perp}$ (MHz)	f_c (MHz)
0.03535	7935.76	7866.96	68.80	7938.2	7868.0	70.2
0.0707	8134.90	7860.02	274.88	8149.5	7865.2	284.3
0.1061	8459.72	7835.89	623.83	8505.8	7853.6	652.2

As in the previously considered square waveguide case the energy stored in the unit length of unperturbed waveguide is

$$U_o = \frac{1}{2}\mu \int |H_z|^2\, dS. \tag{27}$$

Substituting for H_z from (24) and ignoring the constant B (since it cancels from all expressions) we obtain

$$U_o = \frac{1}{2}\mu I^2 \frac{\chi^2}{a^2} \int_{-\phi}^{\phi} \int_{o}^{a} r J_1^2\left(\frac{\chi r}{a}\right) \cos^2\theta\, dr\, d\theta. \tag{28}$$

Here we have used the fact that at cut off

$$\frac{\lambda_c \chi}{2\pi a} = 1. \tag{29}$$

Using

$$\int_{\pi}^{\pi} \cos^2\theta\, d\theta = \pi \tag{30}$$

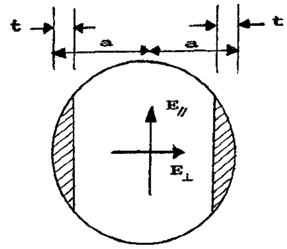


Fig. 5. Circular waveguide polarizer with flats.

(28) becomes

$$U_o = \frac{\pi}{2}\mu I^2 \left(\frac{\chi}{a}\right)^2 \int_o^a r J_1^2\left(\frac{\chi\, r}{a}\right) dr. \tag{31}$$

Making use of the following identity [14]

$$\int x J_1^2(\alpha x)\, dx = \frac{x^2}{2}\left[J_1^2(\alpha x) - J_o(\alpha x)\cdot J_2(\alpha x)\right] \tag{32}$$

then we find

$$U_o = \frac{\pi}{2} I^2 \left(\frac{\chi}{a}\right)^2 \frac{a^2}{2}\left[J_1^2(\chi) - J_o(\chi)\cdot J_2(\chi)\right] \tag{33}$$

which with $\chi = 1.841$ becomes

$$U = \frac{\pi}{4} I^2 \chi^2 \cdot 0.238. \tag{34}$$

A. Calculation of the E Mode Stored Energy and Cutoff Wavelength

Now consider the $E_{\|}$ mode configuration of Fig. 5. In the region of the perturbing flats $\theta \sim 0$, and $J_1'(\chi r/a) = J_1'(\chi) = 0$, so that the dominant field component is H_z. Hence

$$\Delta U_{\|} = \frac{1}{2}\mu |H_z|^2 \Delta V$$
$$= \frac{1}{2}\mu I^2 \left(\frac{X}{a}\right)^2 J_1^2(\chi) \int \cos^2\theta\, d\theta. \tag{35}$$

Note that $J_1(\chi r/a)$ is taken as a constant equal to $J_1(\chi)$ over the perturbed area, an excellent approximation for small flats.

The $\cos^2$ integral is taken over the surface area of the flat and uses the integration scheme shown in Fig. 6. The area of the shaded portion is

$$S = \frac{1}{2} a \cdot a d\theta - \frac{1}{2}\frac{a-t}{\cos\theta} \cdot \frac{a-t}{\cos\theta}\, d\theta \tag{36}$$

so that the required integral becomes

$$\int \cos^2\theta\, d\theta = \frac{1}{2} a^2 \int_{-\phi}^{\phi} \left[\cos^2\theta - (1 - t/a)\right] d\theta. \tag{37}$$

From Fig. 6 we see that

$$\cos\phi = 1 - t/a \tag{38}$$

and

$$\int \cos^2\theta\, d\theta = \frac{1}{2} a^2 \left[\frac{1}{2}\sin 2\phi - \phi\cos 2\phi\right]. \tag{39}$$

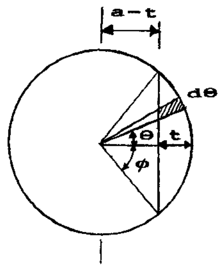


Fig. 6. Integration scheme.

Substituting (39) into (35) gives the final formula for $\Delta U_{||}$ as

$$\Delta U_{||} = \frac{1}{4} N_f \mu I^2 \chi^2 J_1^2(\chi) \left[\frac{1}{2}\sin 2\phi - \phi \sin 2\phi\right] \tag{40}$$

where N_f takes the value of 1 for a single flat on one surface only, and is equal to 2 for two such flats on opposite surfaces.

The shift in resonant frequency is obtained by substitution of (34) and (40) into (7) to give

$$\frac{\Delta f_{||}}{f_o} = 0.225 N_f \left[\frac{1}{2}\sin 2\phi - \phi \cos 2\phi\right]. \tag{41}$$

In terms of cut-off frequency f_o and its shift $\Delta f_{||}$ due to the perturbation, we use results similar to (15–18), i.e.,

$$\frac{\Delta\lambda_c}{\lambda_c} = 1 - \frac{\Delta f_{||}}{f_o}. \tag{42}$$

In order to avoid problems due to uncertainty over the sign of the stored energies we merely recall that the $E_{||}$ mode must have a reduced λ_c and $E_\perp$ an increased λ_c. Hence the final result for the $E_{||}$ mode shift in cut-off wavelength is

$$\frac{\lambda_{c_{||}}}{\lambda_c} = 1 - 0.225 N_f \left[\frac{1}{2}\sin 2\phi - \phi \cos 2\phi\right] \tag{43}$$

where as in (38), $\phi = \cos^{-1}(1 - t/a)$, and the unperturbed cut-off wavelength f or the TE_{11} mode is given as $\lambda_c = 3.412\ a$.

B. Calculation of the $E_\perp$ Mode Stored Energy and Cutoff Wavelength

Here the regions of the perturbing flats correspond to $\theta \sim \pi/2$ in (24), and the dominant fields are E_r and H_θ, which are identical in form apart from the V and I terms. It is also convenient to rotate the fields through 90° so that sin becomes cos and the flats remain at the same angular coordinates as in the $E_{||}$ case.

The stored energy displaced by the flats is then

$$U = \frac{1}{2}\mu |H_\theta|^2 \cdot dS$$
$$= \mu I^2 \frac{J_1^2(\chi)}{a} \int \cos^2\theta \, d\theta. \tag{44}$$

Here, as in the parallel case, $J_1^2(\chi r/a)$ is taken as $J_1(\chi)$ for small flats, and (44) becomes

$$\Delta U_\perp = N_f \mu I^2 \frac{J_1^2(\chi)}{a^2} \frac{1}{2} a^2 \left[\frac{1}{2}\sin 2\phi - \phi \cos 2\phi\right] \tag{45}$$

where we have used the result of (39), and N_f is the number of flats. Substituting (34) and (45) into (7) gives

$$\frac{\Delta U_\perp}{2U_o} = \frac{N_f \mu I^2 J_1^2(\chi)\left[\frac{1}{2}\sin 2\phi - \phi\cos 2\phi\right]}{2\frac{\pi}{4}\mu I^2 \chi^2 \cdot 0.238} \tag{46}$$
$$= 0.1334\, N_f \left[\frac{1}{2}\sin 2\phi - \phi \cos 2\phi\right]. \tag{47}$$

Using the reasoning previously oulined for the $E_{||}$ case, the final result for $E_\perp$ becomes

$$\frac{\lambda_{c_\perp}}{\lambda_c} = 1 + 0.1334\, N_f \left[\frac{1}{2}\sin 2\phi - \phi \cos 2\phi\right] \tag{48}$$

where $\lambda_c = 3.412\ a$.

The results of (43) and (48) are shown in Fig. 7 in comparison with the numerical results of [12, Fig. 5]. It is seen that they are in excellent agreement for small flats where the perturbation theory is particularly applicable. The negligible deviations are acceptable for determination of the required coupling coefficients in the filter case using (6) or (9).

V. Comparison of the Basic Filter Polarizability Theory With HFSS Results

The previous HFSS and other numerical results were applied to 2-dimensional waveguide cross sections. Here the HFSS is applied in 3 dimensions to test the basic relationship between waveguide polarizability and filter coupling coefficient as expressed by (6) or (9). The parameters of the 4th order 2-cavity test filter are given in Table II, and the HFSS analysis displayed a substantially ideal filter response. The object of the test is to confirm the theory for predicting the flat dimension where $b/a = 0.05763$ and for which the coupling coefficient $k = 0.003282$.

Application of (19) and (23) gives

$$\lambda_{c_{||}}/\lambda_c = 0.986859 \quad ; \quad \lambda_{c_\perp}/\lambda_c = 1.00054$$

i.e., a difference of $\Delta\lambda_c/\lambda_c = 0.013195$. From (9) we derive the theoretical value of the coupling coefficient due to the flats as

$$k = (\Delta\lambda_c/\lambda_c) \cdot (\lambda/\lambda_c)^2 = 0.013195 \times (0.993869/2.00)^2$$
$$= 0.003258.$$

This demonstrates almost exact agreement with the HFSS result above of 0.003282.

A similar study was carried out for the circular cross sections with flats. The HFSS study was more difficult to perform due to difficulties in matching the circular boundary to the HFSS discretization, but the results were reasonably good.

VI. Conclusion

A simple formula, (6) or (9), relating the coupling coefficient between the orthogonal modes in a dual mode cavity to the polarizability of the waveguide forming the cavity has been obtained. The polarization is obtained by cutting away portions of the waveguide cavity to form flat regions, and perturbation theory has been applied to relate the polarizability and hence

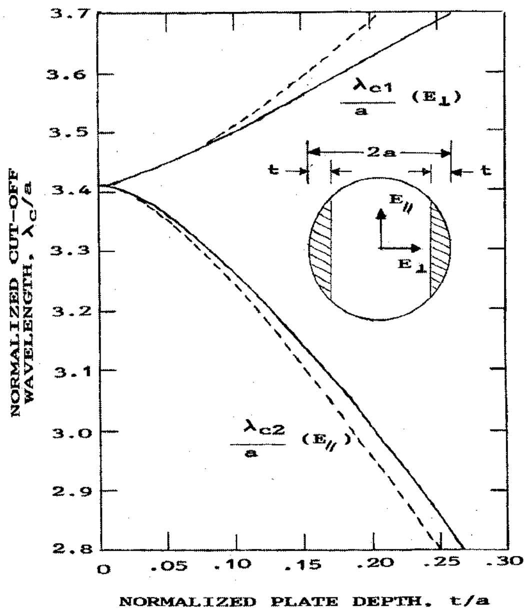


Fig. 7. Curves of cut-off wavelengths of perpendicular and parallel polarizations as a function of flat depth t, with coordinates normalized to the waveguide radius.

TABLE II
PARAMETERS OF TEST FILTER

n = 4, no. dual mode cavities = 2, no. finite frequency poles = 2

Center frequency 11,875 MHz

Bandwidth 38.4 MHz Fractional bandwidth = 0.00323684

Return loss 26.4 dB

Bandwidth at pole frequencies at 2.1 x BW = 80.64 MHz

Coupling coefficients:

Nodes	Normalized	Denormalized
01 = 45	1.16621	0.003771
12 = 34	1.01487	0.003282
23	0.87249	0.002821
14	-0.24628	-0.0007964

Input Waveguide: WR75, 0.750 x 0.375 in.

Resonators: TE_{103}-mode, cross section 1.000 x 1.000 in.
Corner cuts: 0.05763 in. along diagonal
Cavity length: 1.6995 in.

the coupling coefficient to the dimensions in a very simple and direct way. This enables all previous theories relating to waveguide polarizers to be applied to the realization of cross coupling in dual mode cavity filters. The theory has been confirmed by numerical field analysis.

Similar results have been obtained for dual mode dielectric-loaded resonator filters where the dielectric is the full length of the cavity [8].

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Ralph Levy (SM'64–F'73) received the B.A. degree in physics from Cambridge University, England in 1953, the M.A. degree in 1957, and the Ph.D. in applied sciences from London University in 1966.

From 1953 to 1959, he was a member of the Scientific Staff at GEC, Stanmore, England. Where he worked on guided missile, radar, and countermeasures systems, and on waveguide components. In 1959 he joined Mullard Research Laboratories, Redhill, Surrey, England, and continued his work on microwave components and systems, utilizing coaxial and stripline media. He developed a widely-used technique for accurate instantaneous frequency and/or bearing measurement using several microwave discriminators in parallel known as Digital IFM. This ECM work included the development of decade bandwidth directional couplers and broadband matching theory applied to amplifiers. From 1964 to 1967 he was a member of the faculty at Leeds University and carried out research in microwave network synthesis, including realizations of distributed elliptic function filters, and exact synthesis techniques for branch guide and multi-aperture directional couplers. During this period he also consulted for GEC, Decca Radar, and Weinschel Engineering. From 1967 until 1984 he was with Microwave Development Laboratories, Natick, MA, as Vice President of Research. His work there resulted in practical techniques for designing very broad band mixed lumped and distributed circuits, such as the tapered corrugated waveguide harmonic rejection filter, and the synthesis of a variety of microwave passive components. These included the development of multi-octave multiplexers in suspended substrate stripline, requiring accurate modeling of various inhomogeneous stripline circuits and discontinuities. From 1984 to 1988 he was with KW Microwave, San Diego, CA., as Vice President of Engineering, working mainly on design implementations and improvements in their filter based products. From 1988 to 1989 he was with Remec Inc., San Diego, CA. as Vice President, and continued with advances in suspended substrate stripline components, synthesis of filters with arbitrary finite frequency poles, and microstrip filters. In July 1989 he became an independent Consultant and has worked with several companies on a wide variety of projects, mainly in the field of passive components, ranging from high-power waveguide components to miniature filters and multiplexers. He is the author of more than 60 papers, 2 books, and 12 patents.

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